

A. Mathematical modeling

We consider the fittest solution can be described as the alpha (α) followed by the second and third best solutions which are beta (β) and delta (δ) respectively. The other candidate solutions are considered as omega (ω). GWO has set the hunting (optimization) is guided by α , β and δ while the ω wolves just following them. During the wolves hunting, they encircle their prey and the following equations described their encircling behavior [24]:

$$\vec{D} = |\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)| \quad (12.a)$$

$$\vec{X}(t + 1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D} \quad (12.b)$$

Where, t is the current iteration, \vec{A} and \vec{C} are coefficient vectors, \vec{X} is the position vector of a grey wolf, and \vec{X}_p is the position vector of the prey. The vectors \vec{A} and \vec{C} are calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (13.a)$$

$$\vec{C} = 2 \cdot \vec{r}_2 \quad (13.b)$$

Where components of \vec{a} are linearly decreased from 2 to 0 over the course of iterations and r_1, r_2 are random vectors in [0,1].

The three best solutions are saved and then the other search agents (ω -wolves) update their positions according to the current best position. These situations are expressed in the following expressions:

$$\vec{D}_\alpha = |\vec{C}_1 \cdot \vec{X}_\alpha(t) - \vec{X}(t)| \quad (14.a)$$

$$\vec{D}_\beta = |\vec{C}_2 \cdot \vec{X}_\beta(t) - \vec{X}(t)| \quad (14.b)$$

$$\vec{D}_\delta = |\vec{C}_3 \cdot \vec{X}_\delta(t) - \vec{X}(t)| \quad (14.c)$$

$$\vec{X}_1 = \vec{X}_\alpha(t) - \vec{A}_1 \cdot \vec{D}_\alpha \quad (15.a)$$

$$\vec{X}_2 = \vec{X}_\beta(t) - \vec{A}_2 \cdot \vec{D}_\beta \quad (15.b)$$

$$\vec{X}_3 = \vec{X}_\delta(t) - \vec{A}_3 \cdot \vec{D}_\delta \quad (15.c)$$

$$\vec{X}(t + 1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \quad (16)$$

Equations (14.a), (14.b), and (14.c) estimate the distance between α , β and δ with respect to ω respectively. Then equations (15.a), (15.b), and (15.c) decide the current position of α , β and δ respectively. Final equation (16) indicates to the update position of ω depending on the positions of the best three solutions (α , β and δ).

IV. IMPLEMENTATION OF PROPOSED GWO ALGORITHM

Several steps have been taken to implement GWO to obtain the optimal allocation (i.e. site and size) of multi-DG units. This procedure is described through the flowchart presented in Fig.3 [20]. The pre-decided maximum number of iterations (itmax), the dimension (dim) of the problem and the search agents (NSA) are applied.

• Step 1: Initialization

Firstly, the itmax, NSA, dim and constraints of the problem are initialized.

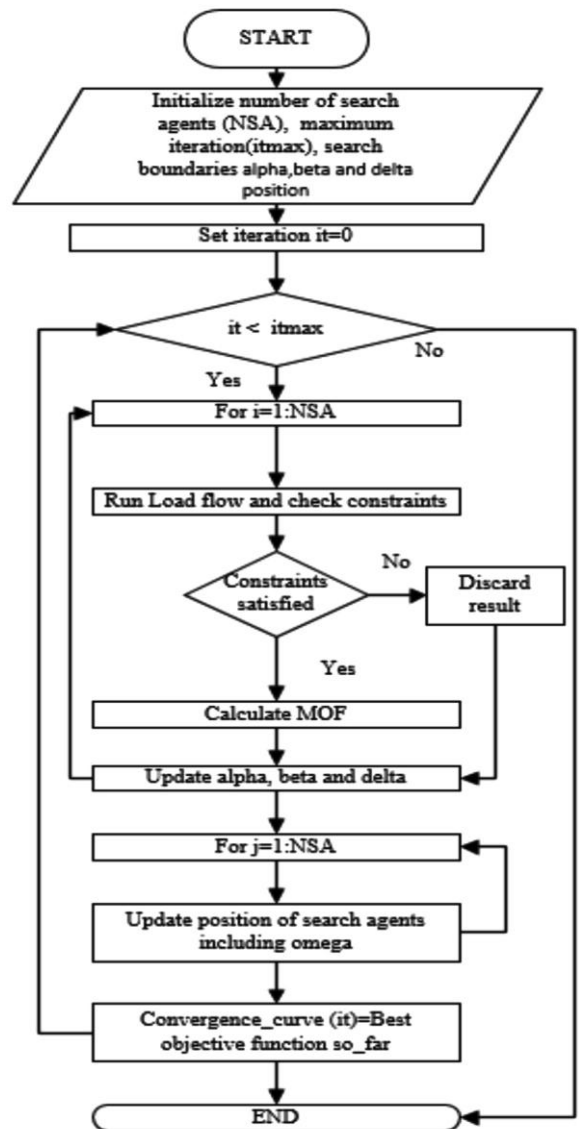


Fig. 3. The flow chart of the proposed GWO algorithm [20].

• Step 2: Generation of grey wolf positions

A population of grey wolves is generated by GWO and α , β , and δ wolves' positions are initialized and then the objective function for each population is calculated through load flow method.

• Step 3: Quality solution

The constraints of each search agent are checked and if the constraints are satisfied, then the multi-objective function is

calculated. But in the case of any constraint violation, the results are discarded.

• **Step 4: Choose the best position**

The positions of α , β , and δ wolves are updated, except the ω wolf and then updating ω wolf by utilizing Eqns. (5)-(10) in order to determine the best solution so far.

• **Step 5: Calculation of new positions of search agents**

The new positions of the search agents are determined and the whole process is repeated.

• **Step 6: Termination**

In the proposed study, the stop criteria are set as maximum iterations. When the criterion is satisfied, then the simulation will be stopped and the optimum site and size of multi-DG units which satisfy all the specified constraints of the distribution system will be obtained.

V.SIMULATION RESULTS AND DISCUSSION

The simulation results will be carried out on two different distribution systems which are IEEE 33-bus and IEEE 69-bus test radial distribution systems. For both test systems, it is assumed that all buses of the network can be taken as candidate for DG units' placement except the first bus which connected to the main feeder from generation station. DG units will be considered as photo-voltaic cells ($pf=1$) generate only active power. To evaluate the effectiveness of the proposed GWO algorithm, the performance of the systems are analyzed and compared with GA optimization method. GWO is used to obtain the optimal number, size and location of DG units for both test systems.

A. IEEE 33-bus test system

This system consists of 33 bus and 32 line with a feeder connected to bus 1 as shown in Fig. 4 and data is given in [26]. The total active and reactive power loads of the system are 3.715 MW and 2.3 MVar, respectively. The total real power loss is 210.99 kW and the total reactive power loss is 143.12 kVar with the weakest voltage point of 0.9092 pu at bus (18) which are calculated by load flow method mentioned in section II.

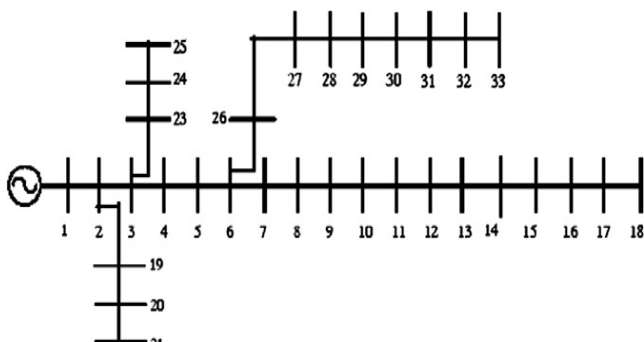


Fig. 4. Single line diagram of IEEE 33-bus test system.

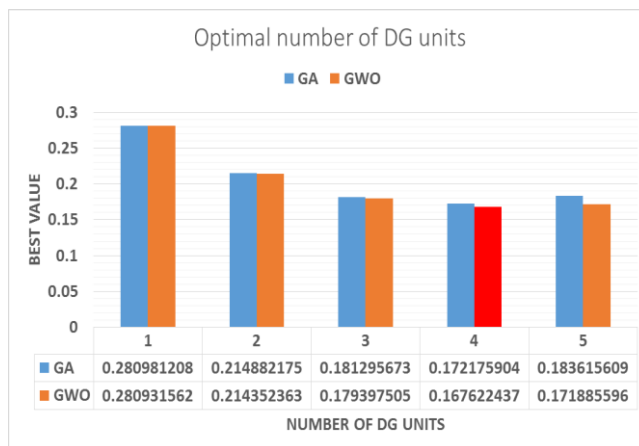


Fig. 5. Optimal number of DG units for 33 bus test system.

Fig.5 shows a comparison between GA and GWO techniques in terms of the best values of the multi-objective function to obtain the optimal number of DG units in IEEE 33-bus test system. Based on Fig.5, the optimal number of DG units is four units since in case of five units the best value of the multi-objective function is more than that obtained in case of four units under the specified constrains.

Table I Comparison of IEEE 33-bus test system for one DG unit with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.285824129	0.280981208	0.297648638
GWO	0.280931565	0.280931562	0.280931581

Table II Comparison of IEEE 33-bus test system for two DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.22004968	0.214882175	0.2386613
GWO	0.214353025	0.214352363	0.214776908

Table III Comparison of IEEE 33-bus test system for three DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.196705658	0.181295673	0.210195473
GWO	0.179656092	0.179397505	0.180567979

Table IV Comparison of IEEE 33-bus test system for four DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.186605426	0.172175904	0.20817918
GWO	0.173157877	0.167622437	0.183269432

Tables I-IV show the comparison between GA and GWO in terms of average, best and worst values. The difference between the average and best values with GWO and GA shows the convergence of solutions in case of GWO than GA. On the other hand, difference between the best and worst values with GWO and GA reflects the accuracy and superiority of GWO over GA.

Fig. 6 to Fig. 9 show the convergence curves of the GWO for the best solution in case of single and multi-DG units for IEEE 33-bus system. It is clear from the convergence curves that GWO is a sufficient optimization technique as it is able to find the best solution even if the number of variables is increasing.

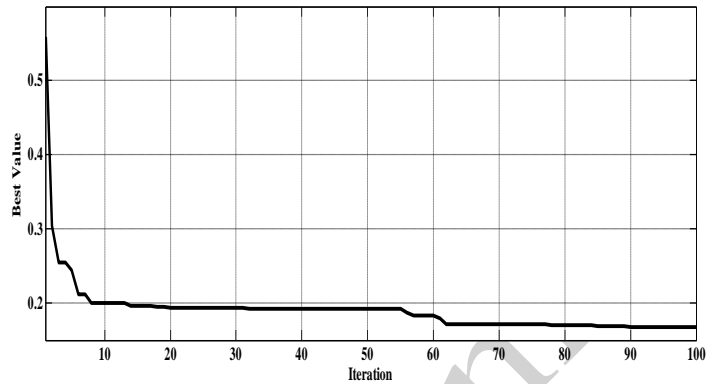


Fig. 9. Convergence curve of GWO for the best solution of three DG units.

Fig.10 shows a comparison between the voltage levels at all buses for IEEE 33-bus test system in case of base case (without DG units), single and multi-DG units using GWO.

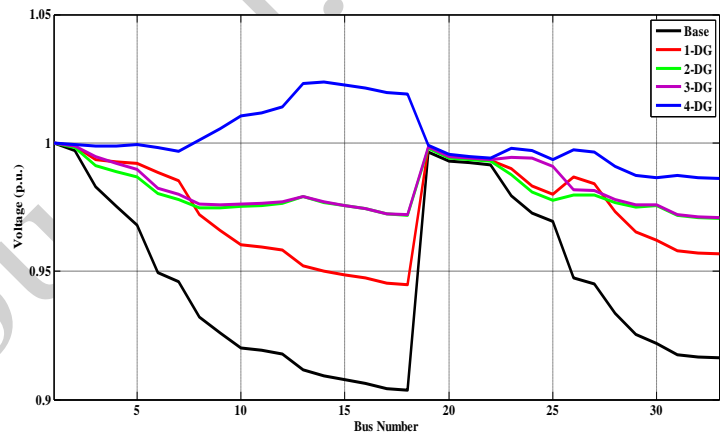


Fig. 10. Comparing voltage profile for single and multi-DG units of IEEE 33-bus test system.

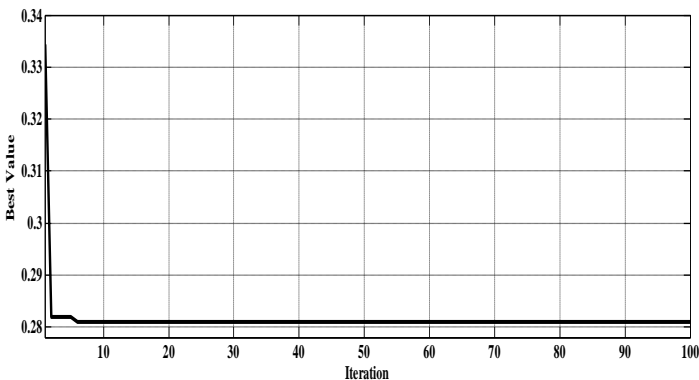


Fig. 6. Convergence curve of GWO for the best solution of one DG.

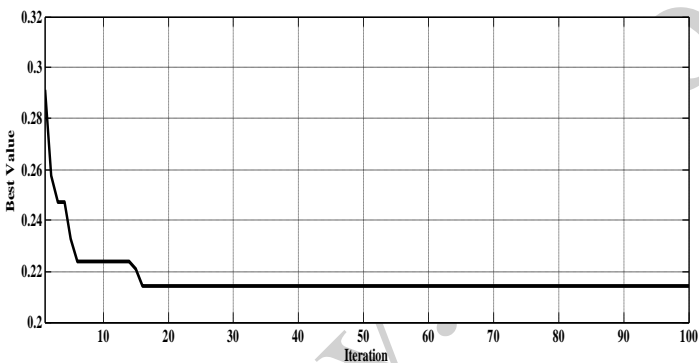


Fig. 7. Convergence curve of GWO for the best solution of two DG units.

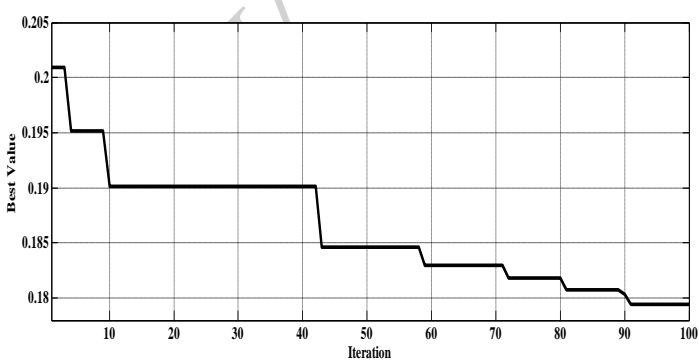


Fig. 8. Convergence curve of GWO for the best solution of three DG units.

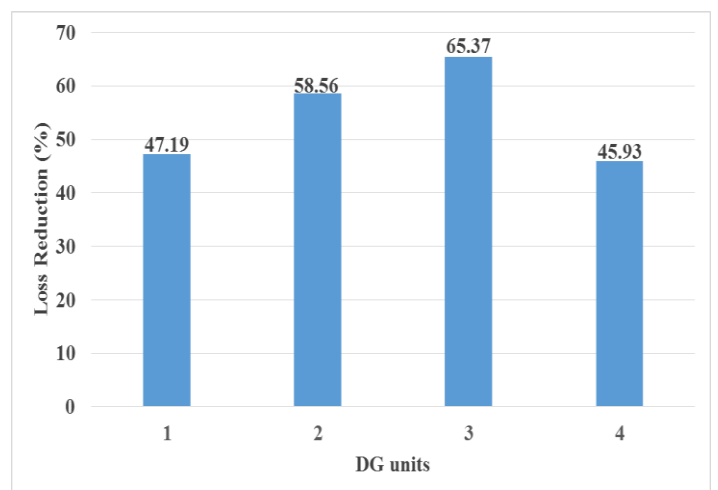


Fig. 11. Comparing power loss reduction (%) for single and multi DG units of 33-bus test system.

Table V Comparing single and multi DG units using GWO of IEEE 33-bus test system performance.

Using GWO	Power Losses (kw)	Minimum Voltage (pu)	Weakest Bus	DG Location	DG size (KW)	Total DG (KW)
Base Case	210.99	0.9092	Bus (18)	----	----	----
One DG	111.42	0.9448	Bus (18)	Bus (6)	2761.82	2761.82
Two DG	87.43	0.9706	Bus (33)	Bus (13) Bus (30)	903.04 1201.61	2104.65
Three DG	73.06	0.9709	Bus (33)	Bus (13) Bus (24) Bus (30)	850.02 1103.87 1100.77	3054.66
Four DG	114.08	0.9862	Bus (33)	Bus (6) Bus (14) Bus (24) Bus (31)	797.81 785 971.9 785	3339.71

According to Fig. 10, the system needs at least two DG units to improve the voltage level of all buses more than 0.95 pu which is the minimum accepted limit. With only one DG unit there is a weak point at bus (18) at which the voltage level is lower than 0.95 pu. The voltage levels in case of four DG units is better than that in case of two and three DG units.

Fig. 11 shows a comparison between power loss reduction in a percent with respect to the base value for single and multi-DG units using GWO. It is clear that, the loss reduction in case of three units is better than the other cases. Although increasing the number of DG units should reduce the power losses, the power losses are increased in case of four DG units for the sake of better voltage levels.

Table V shows a comparison between all study cases in terms of the best size, location, power loss and weakest bus for IEEE 33-bus test system using GWO.

Based on the above results, three DG units is the most suitable and recommended case for this system. Although four DG units give the best overall multi-objective function, but it is recommended to use only three DG units for economic point of view, best power loss reduction, and the voltage levels are within the minimum and maximum accepted voltage limits.

B. IEEE 69-bus test system

The second system is IEEE 69-bus test radial distribution system which has 69 bus and 68 line with a source at bus 1 as demonstrated in Fig. 12. It has the total load of 3.80 MW and 2.69 MVar and its data for this system are given in [26]. The real power loss and the reactive power loss are 224.9 KW and 102.13 KVar for this test system respectively with the weakest voltage point of 0.9092 pu at bus (65) which are calculated by load flow method mentioned in section II.

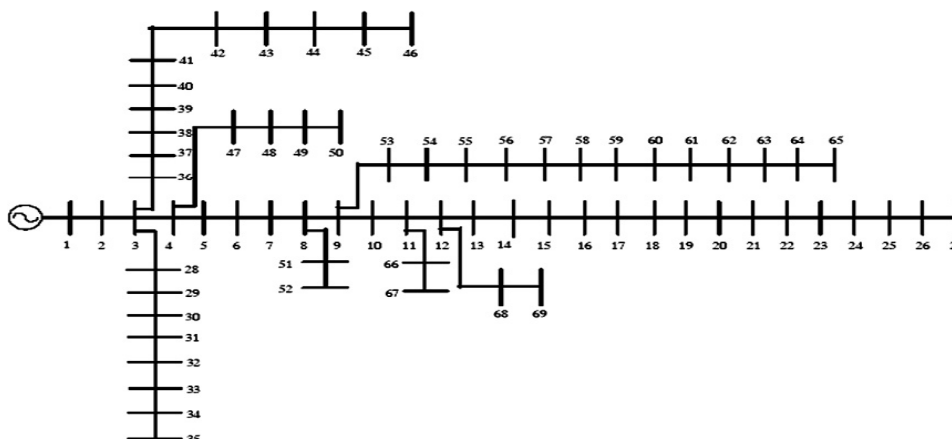


Fig. 12. Single line diagram of IEEE 69-bus test system.

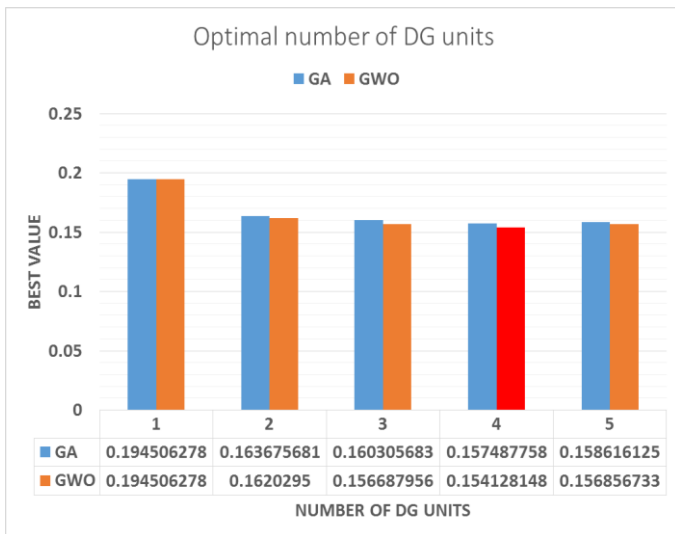


Fig. 13. Optimal number of DG units for IEEE 69-bus system.

Fig.13 shows a comparison between GA and GWO techniques in terms of the best values of the multi-objective function to obtain the optimal number of DG units in IEEE 69-bus test system. Based on Fig.13, the optimal number of DG units is four units since in case of five units the best value of the multi-objective function is more than that obtained in case of four units under the specified constrains.

Table VI Comparison of IEEE 69-bus test system for one DG unit with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.196613258	0.194506278	0.203292237
GWO	0.194506292	0.194506278	0.194506315

Table VII Comparison of IEEE 69-bus test system for two DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.182066859	0.163675681	0.203157877
GWO	0.165167585	0.1620295	0.170473961

Table VIII Comparison of IEEE 69-bus test system for two DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.166414764	0.160305683	0.179906007
GWO	0.159697218	0.156684205	0.170909983

Table IX Comparison of IEEE 69-bus test system for two DG units with 30 different trials.

Method	Average	Best solution	Worst solution
GA	0.166468793	0.157487758	0.187069066
GWO	0.156334565	0.154124338	0.167496306

Tables VI-IX show the comparison between GA and GWO in terms of average, best and worst values which verified the superiority of GWO over GA as executed in case of IEEE 33-bus test radial distribution system.

Fig. 14 to Fig. 17 show the convergence curves of the GWO for the best solution in case of single and multi-DG units for IEEE 69-bus system.

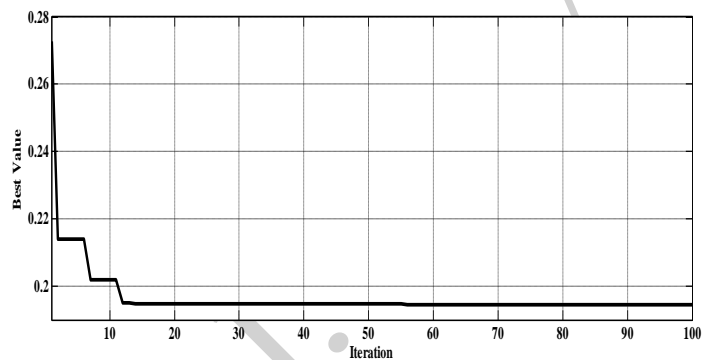


Fig. 14. Convergence curve of GWO for the best solution of one DG units in IEEE 69-bus test system.

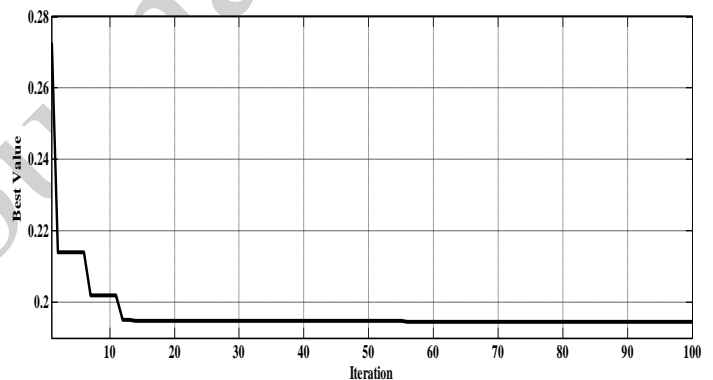


Fig. 15. Convergence curve of GWO for the best solution of two DG units in IEEE 69-bus test system.

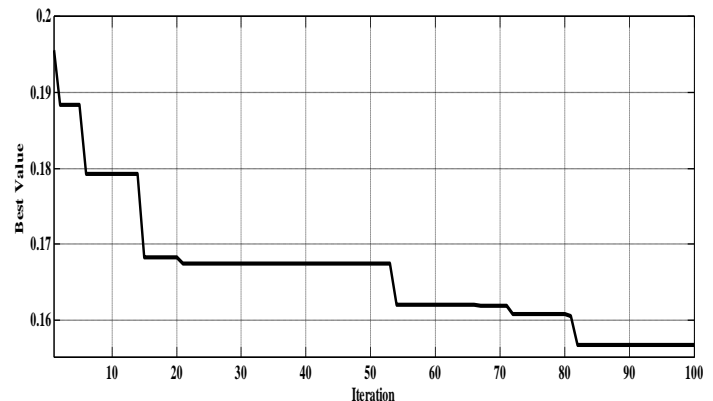


Fig. 16. Convergence curve of GWO for the best solution of three DG units in IEEE 69-bus test system.

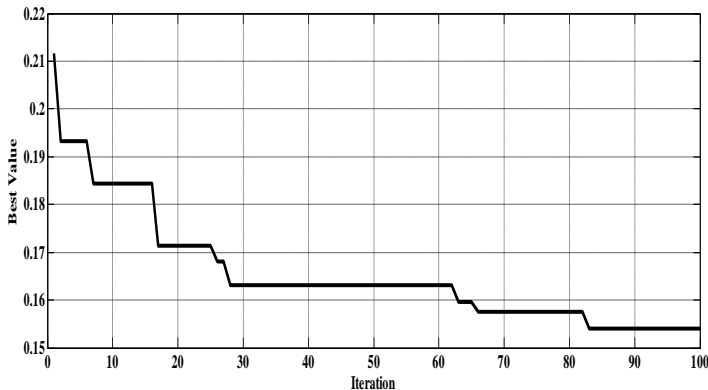


Fig. 17. Convergence curve of GWO for the best solution of four DG units in IEEE 69-bus test system.

Fig.18 shows a comparison between the voltage levels at all buses for IEEE 69-bus test system in case of base case (without DG units), single and multi-DG units using GWO.

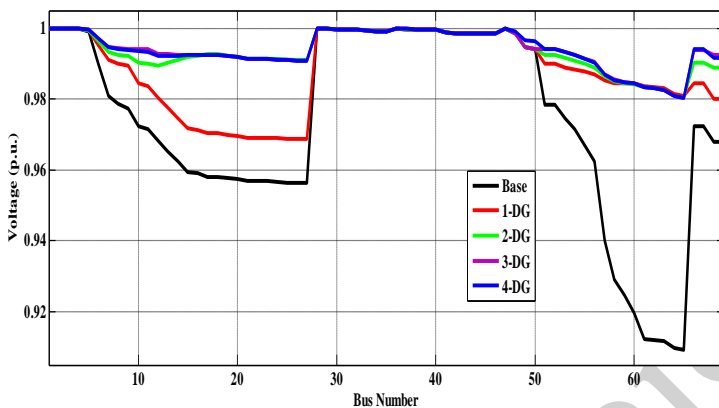


Fig. 18. Comparing voltage profile for single and multi-DG units of IEEE 69-bus test system.

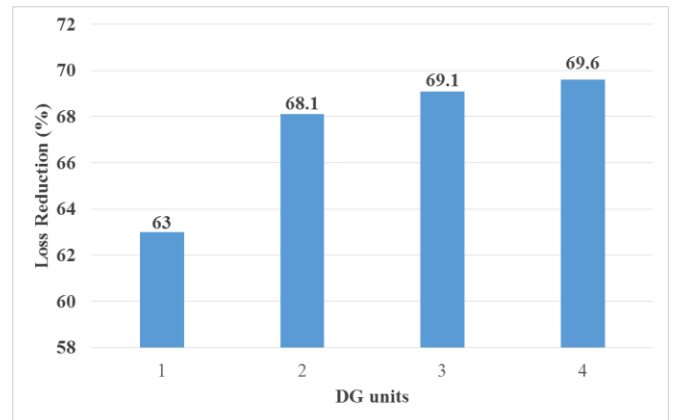


Fig. 19. Comparing power loss reduction (%) for single and multi DG units of IEEE 69-bus test system.

According to Fig. 18, the system needs only one DG unit to improve the voltage level of all buses more than 0.95 pu which is the minimum accepted limit.

Fig. 19 shows a comparison between power loss reduction in a percent with respect to the base value for single and multi-DG units using GWO. It is clear that, the loss reduction in case of four DG units is better than the other units.

Table X shows a comparison between all study cases in terms of the best size, location, power loss and weakest bus for IEEE 69-bus test system using GWO.

The weakest bus is the same for two cases, three, and four DG units but four units case has the lowest power losses. Finally, although four DG units case gives the best overall multi-objective function, but due to the slightly difference between four and three units, we suggest case with only three DG units for less size of DG units used from the economic point of view.

Table V Comparing single and multi DG units using GWO of IEEE 69-bus test system performance.

Using GWO	Power Losses (kw)	Minimum Voltage (pu)	Weakest Bus	DG Location	DG size (KW)	Total DGs (KW)
Base Case	224.9	0.9092	Bus (65)	----	----	----
One DG	83.24	0.9687	Bus (27)	Bus (61)	1928.67	1928.67
Two DG	71.74	0.9803	Bus (65)	Bus (17) Bus (61)	566.08 1816.42	2382.5
Three DG	69.51	0.9804	Bus (65)	Bus (11) Bus (18) Bus (61)	541.52 406.39 1751.43	2699.34
Four DG	68.35	0.9803	Bus (65)	Bus (18) Bus (50) Bus (61) Bus (67)	439.79 716.47 1756.74 473.68	3386.68

VI. CONCLUSION

This paper presents a comparison between two optimization techniques which are GA and GWO. Results show the effectiveness, superiority and accuracy of GWO technique over the GA technique. The GWO is then used to get the best number, size and location of DG units in case of two radial study systems which are IEEE 33-bus test system and IEEE 69-bus test system. Several cases based on number of DG units are studied considering a multi-objective function and then the best number of DG units is recommended based on the results of this multi-objective function and some economic aspects.

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